Elimination of Hot Cracking in Laser Beam Welding

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Hot cracking is one of the big problems in laser beam welding. By multi-beam welding, first suggested in the 70-s (cf. [1], [4]), hot cracking can be avoided. Hereby two additional laser beams are employed. These beams generate a compression which compensate for the critical tensile strain in the solid-liquid region of the weld induced by the main laser beam. However, hot cracking can only be prevented if the positions, sizes, and powers of the additional laser beams are suitably chosen, i.e. are optimized. Non-optimal values can even enhance hot cracking. Until now these quantities have been found either by trial and error or by prescribing them intuitively. In the present paper a constrained nonlinear programming problem is formulated to solve the problem of hot crack initiation by minimizing the accumulated transverse strain, i.e. the opening displacement, in the solid-liquid region. This approach is based on the so-called strip expansion technique, cf. [2].

1 Thermomechanical Model

The accumulated transverse strain (opening displacement) \( W_{od} \) in the solid-liquid region of the weld can be described as follows, cf. [2]:

\[
W_{od} = 2\alpha \int_0^B [T(x_L, y) - T(x_S, y)] \, dy.
\]

Here, \( T \) denotes the temperature depending on the spatial coordinates \( x \) and \( y \), \( \alpha \) and \( B \) denote the thermal expansion coefficient and the distance from the centerline of the weld to the restraint. In order to describe the abscissae \( x_S \) and \( x_L \), the isothermes are considered which are associated with the solidus and liquidus temperature; see the coloured curves in Fig. 1. These isothermes enclose the solid-liquid region in which hot crack initiation takes place. Their points of intersection with the \( x \)-axis behind the main laser beam are the \( x \)-coordinates \( x_S \) and \( x_L \). Hot cracking does not occur when \( W_{od} \) drops below a critical value to be determined experimentally. Thus minimization of the opening displacement prevents hot cracking.

For the description of the temperature field it is therefore sufficient to consider only the quasi-stationary state. This leads to the following quasi-stationary 2D heat equation in a moving reference frame:

\[
\Delta T(x, y) + \frac{v}{a} \frac{\partial T(x, y)}{\partial x} + \frac{\lambda}{\rho s} q_2(x, y) = 0, \quad q_2(x, y) = \begin{cases} q_{add} = \frac{q_{add}}{\pi r_{add}^2} & \text{if } (x-x_{add})^2 + (y-y_{add})^2 \leq r_{add}^2 \\ q_{main} = -\frac{q}{\pi r^2} & \text{if } x^2 + y^2 \leq r^2 \quad (r \ll 1) \\ 0 & \text{otherwise} \end{cases}
\]

with boundary conditions of Dirichlet type: \( T(x, B) = T(x, -B) = T(\pm\infty, y) = T_0 = \text{const.} \).

Hereby the first term in Eq. (1) describes the diffusion of the temperature \( T \). The second term is caused by the moving reference frame due to the moving laser beams. Their travel speed is \( v \). The thermal diffusivity is \( a \) and the last term is a source term where \( q_2(x, y) \) stands for the area-specific heat source density, piecewisely defined as in (1). \( \lambda \) denotes the thermal conductivity and \( s \) the thickness of the plate. Furthermore \( q \) denotes the power of the main laser beam. The additional laser beams both have the power \( q_{add} \) and affect a domain \( D = D_1 \cup D_2 \), consisting of two disjoint circles, in an evenly distributed way. The other quantities with subscript add in the definition of \( q_2 \) will be explained later on. \( T_0 \) denotes the initial temperature. The quantities \( v, a, \lambda, \) and \( s \) are assumed to be constant.

This problem has an analytical solution if the plate is infinite (cf. [3]). In order to fulfill also the boundary conditions, the method of images is used. This yields the unique solution of the elliptical boundary value problem:

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\[
T(x, y) - T_0 = \sum_{i = -\infty}^{\infty} \frac{q}{2\pi \lambda_s} \exp\left(-\frac{v x}{2a}\right)[f(x, y - 2i(2B)) - f(x, y - (2i + 1)(2B))] \\
+ \sum_{i = -\infty}^{\infty} \int_{D} \int_{D} \frac{q_{\text{add}}}{2\pi \lambda_s} \exp\left(-\frac{v(x - \xi)}{2a}\right)[f(x - \xi, y - 2i(2B) - \eta) - f(x - \xi, y - (2i + 1)(2B) + \eta)] \, d\xi \, d\eta.
\]

Here, \( f(\arg_1, \arg_2) := K_0(v ||(\arg_1, \arg_2)||_2 / (2a)) \) with \( K_0(\cdot) \) denoting the modified Bessel function of the second kind of order zero.

### 2 Constrained Nonlinear Optimization

The geometry and the optimization variables are shown in the following figure:

![Top view of the plate](image)

**Fig. 1** Top view of the plate

Now the temperature \( T \) depends on all parameters \( p := (p_1, \ldots, p_6) \). The parameters \( p_5 \) and \( p_6 \) are implicitly defined by the two aforementioned equations, which are taken into account in the optimization process by penalty terms.

Hence the nonlinear programming problem reads as follows:

\[
\min_{p \in \mathbb{R}^6} \left\{ r_1 \left[ \int_{0}^{B} [T(p_0, y; p) - T(p_0, y; \hat{p})] \, dy \right] + r_2 \left[ \left( \frac{(T(p_0, 0; p) - T_L)^2}{T_L^2} + \frac{(T(p_0, 0; p) - T_S)^2}{T_S^2} \right) \right] \right\}, \quad r_1 + r_2 = \text{const}
\]

subject to \( 0.5 \leq p_3 \leq 1200 \), \( 0 \leq p_4 \leq B \), and \( T_{\max}(x, y, p) \leq T_S \) in \( D \). In a first step the maximum temperature in the last constraint is taken just in the point \( (p_1 - p_3, p_2) \). This is the hottest point of a circularly radiated single laser beam.

### 3 Results

Numerical results were computed by an SQP method. The two pictures show the temperature field with and without additional laser beams. The multi-beam technique leads to a more evenly distributed temperature and therefore to a much larger solid-liquid region. Nevertheless the tensile strain of the main laser beam is compensated so that no hot cracking arises.

![No additional heat sources](image)

**Fig. 2a** No additional heat sources

![Additional heat sources; optimal solution](image)

**Fig. 2b** Additional heat sources; optimal solution

### References


