With the vision of the Smart Factory of the future, the manufacturing industries are currently undergoing a fundamental new orientation on the basis of the Cyber-Physical Systems and Internet of Things and Services paradigms. All parts along the manufacturing chain are nowadays equipped with embedded computing, communication and networking capabilities and are expected to interact in an optimal way towards the goal of an energy and resource efficient, save and reliable production process. Through decentralized optimal decision-making and an appropriate communication among the networked individual parts, the whole production process of the future is expected to operate optimally.

In this presentation an introduction to the goals and principles of Industry 4.0 is given and its challenges and opportunities for the field of mathematical control theory are discussed. We will in particular investigate the potential impact of the field of optimization-based control for the fourth industrial revolution and will discuss several promising directions including economic model predictive control and distributed, cooperative optimization and control.

Already a few years after the discovery of the most useful necessary condition of optimal control theory, the maximum principle, by Pontryagin and his students Boltyanski and Gamkrelidze in 1955, Bryson, Denham and Dreyfus suggested a method in 1963 to establish necessary conditions for state constrained optimal control problems which were especially appropriate for numerical implementations.

The talk will exhibit how their idea can be transferred to optimal control problems for partial differential equations. For the sake of simplicity, we will focus on elliptic optimal control problems with state constraints and we will see that these problems can be reformulated as topology, resp. shape optimization problems with the interface between active and inactive set as optimization variable. This type of problems leads to a new class of so-called set optimal control problems which can also be interpreted as bi-level optimization problems, more abstract as optimization problems on vector bundles.