On the Mutual Impact of Systems Governed by ODEs and PDEs in Optimal Control with Applications to Hypersonic Flight of Civil Aircraft

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Abstract

During ascent and reentry of a hypersonic space vehicle into the atmosphere of any heavenly body, the space vehicle is subjected, among others, to extreme aerothermic loads. Therefore an efficient, sophisticated and lightweight thermal protection system is determinative for the success of the entire mission. For a deeper understanding of the conductive, convective and radiative heating effects through a thermal protection system, a mathematical model is investigated which is given by an optimal control problem subject to not only the usual dynamic equations of motion and suitable control and state variable inequality constraints, but also to an instationary quasi-linear heat equation with nonlinear boundary conditions. By this model the temperature of the heat shield can be limited in certain critical regions.

The resulting ODE-PDE constrained optimal control problem is solved by a second-order semi-discretization in space of the quasi-linear parabolic partial differential equation yielding a large scale nonlinear ODE constrained optimal control problem with additional state constraints for the heat load. Numerical results are presented. The aerothermic load can be sufficiently limited by optimization.

This real-life problem motivates a new class of optimal control problems with constraints in form of a coupled system of ordinary and partial differential equations. For this class, we present a subclass of academic examples which reveal the complicated theoretical background of state-constrained ODE-PDE optimal control problems caused by their mutual impact. We call this subclass of problems, hypersonic rocket car problems, since it is inspired, on the one hand, by the well-known rocket car problem, which often serves as a propaedeutic example in courses of optimal control, and, on the other hand, by the abovementioned flight path trajectory optimization problem for a hypersonic aircraft.
Moreover, these problems can be considered as undressed abstract examples for a class of staggered state-constrained ODE-PDE-constrained optimal control problems that are typical for many current applications. The simplification allows to obtain analytical solutions to a certain extend which is normally prohibited by the enormous complexity of real-life problems.

Firstly, the analysis of structural questions concerning the existence of boundary arcs and touch points of state constraints is the aim of the talk. This is novel in the context of PDE-constrained optimal control. We obtain results, which are, at a first glance, similar to state-constrained ODE optimal control problems and show their relation to the differentiation index of the related partial differential algebraic equation system along state-constrained subarcs. At a second glance, new phenomena are observed caused by the non-local character of the state constraint in the ODE context. A crucial point namely is that the state constraint, which is pointwisely defined in the PDE context, loses its local character from the ODE point of view. This leads to new types of necessary conditions, both for the ODE and the PDE formulations of the hypersonic rocket car problems.

In any case, integral relations between variables are present in certain necessary conditions making the application of adjoint based methods a challenge, if not impossible. In particular, new necessary conditions for unspecified terminal time and new jump conditions for certain state-constrained parabolic control problems are developed.

Comprehensive studies concerning the two essential numerical solution approaches, first discretize, then optimize (direct method) and first optimize, then discretize (indirect method) are carried out. It seems that only the first method can be performed with a passable effort. Nevertheless, many of the necessary conditions can be at least approximately verified on the basis of adjoint estimates from the direct method.